CAACS: A *Carbon Aware* Ant Colony System

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MIT PRIMES Conference

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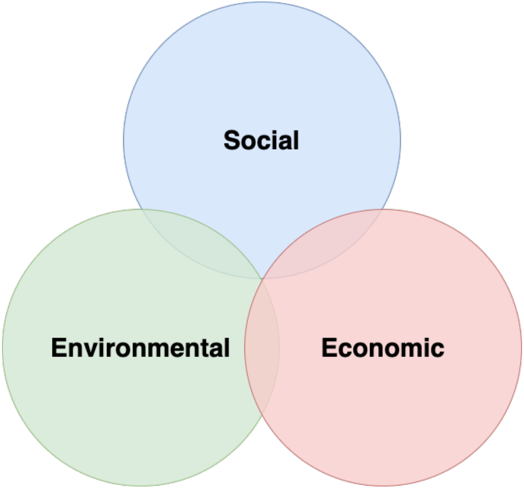
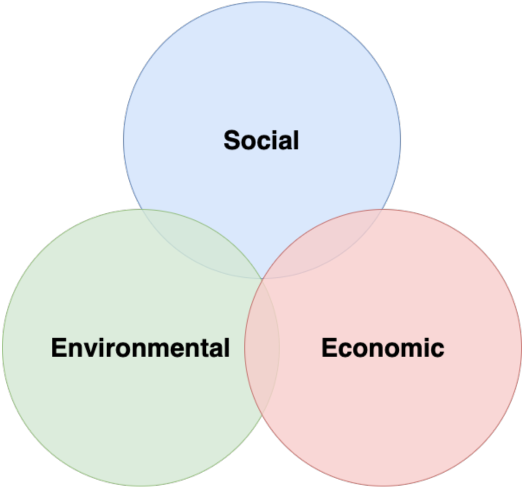
Sustainability



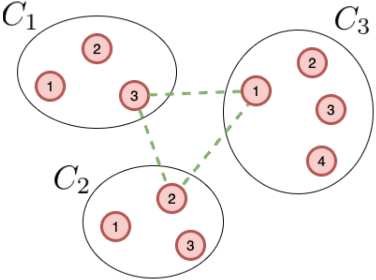
Sustainability



How can we balance multiple objectives of sustainability?

Generalized Traveling Salesman Problem (GTSP)

Problem Statement: Consider an undirected graph $G = (V, E)$ where the vertex set V is partitioned into n distinct clusters $C_1, C_2, \ldots, C_n$. The GTSP searches for the shortest Hamiltonian cycle such that exactly one vertex from each cluster is visited.



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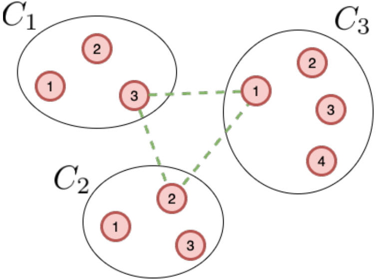
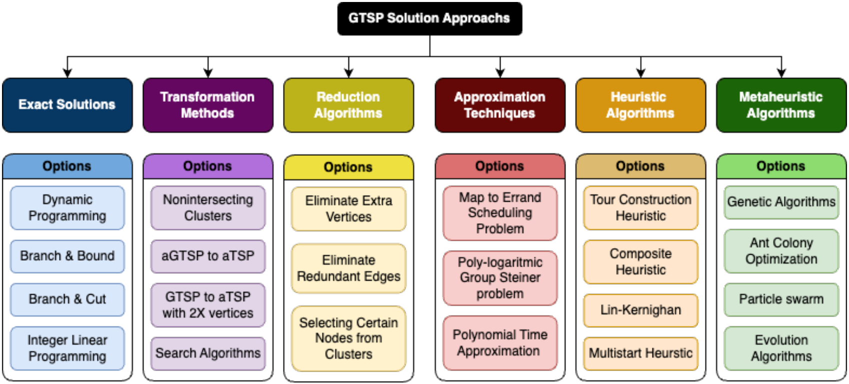
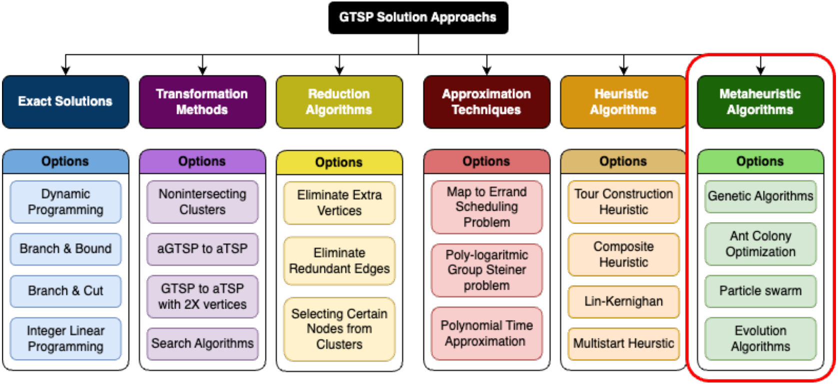


Applications: Logistics, Microchip Design, UPS Package Delivery, Medical Supplies Distribution, and a Subproblem of DNA Sequencing!

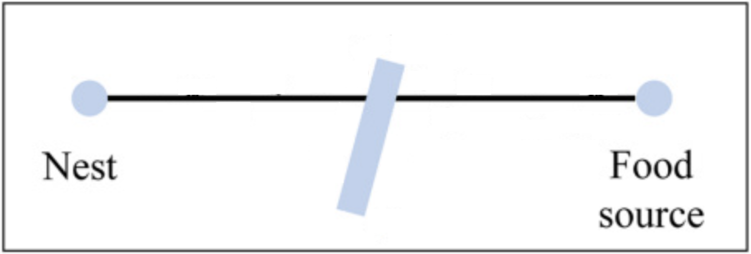
Solutions to GTSP



Solutions to GTSP



Ant Colony Optimization



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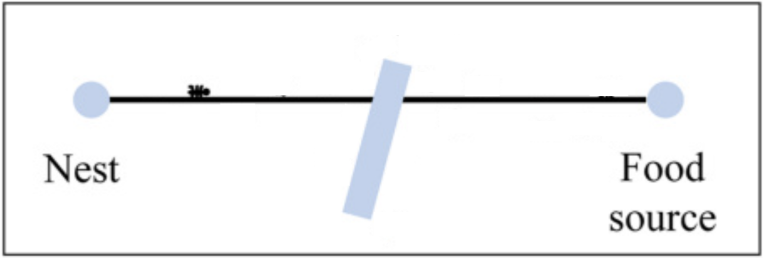

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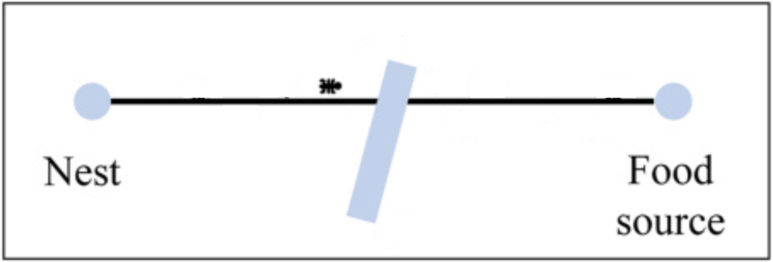

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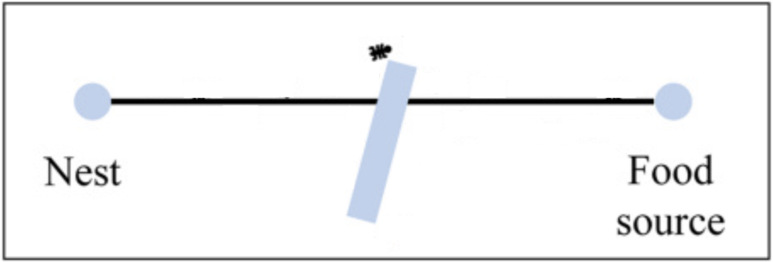

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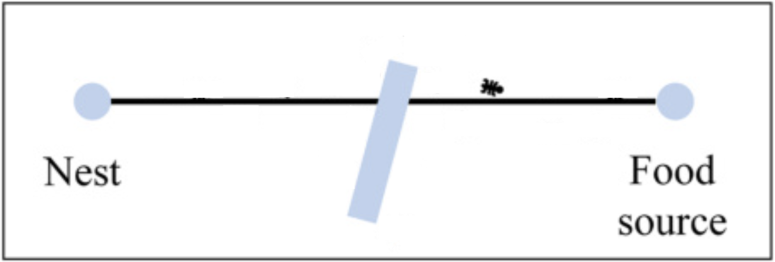

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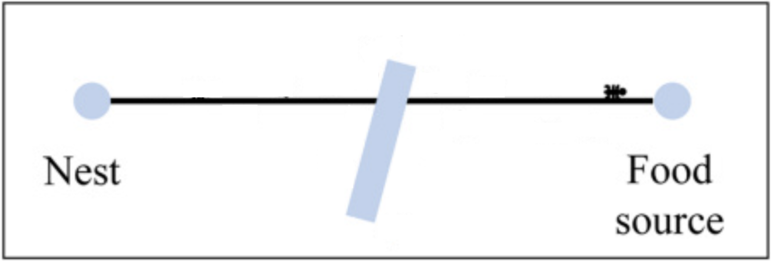

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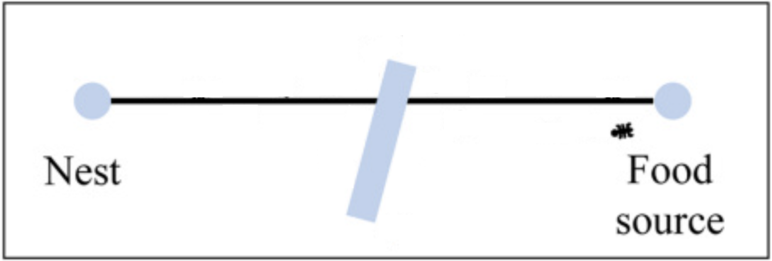

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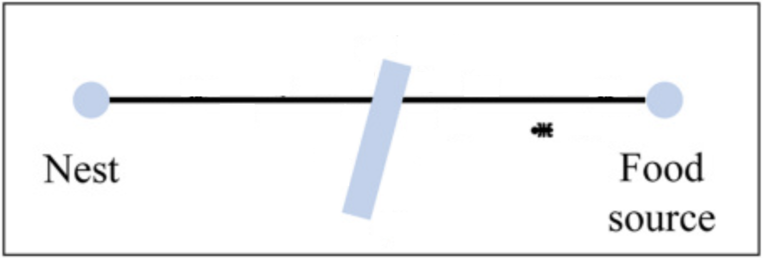

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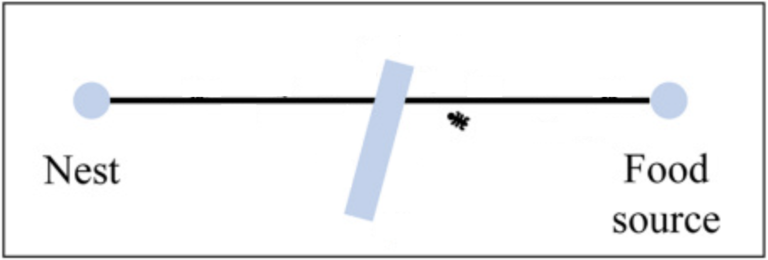

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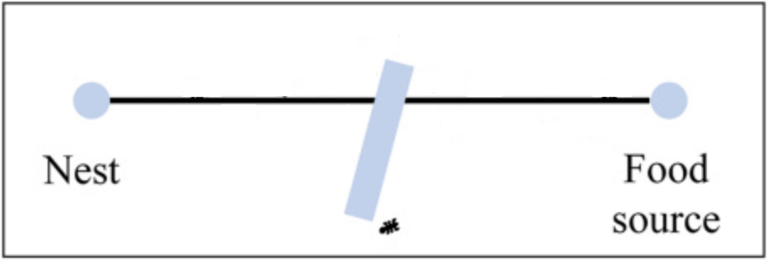

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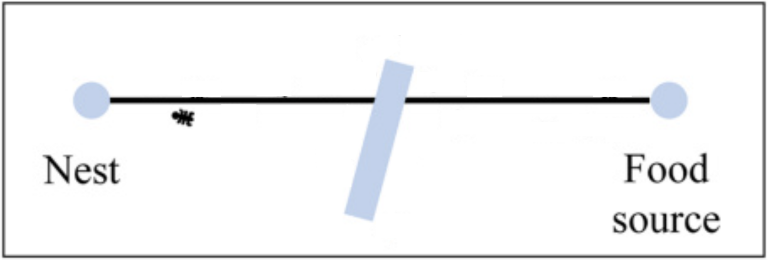

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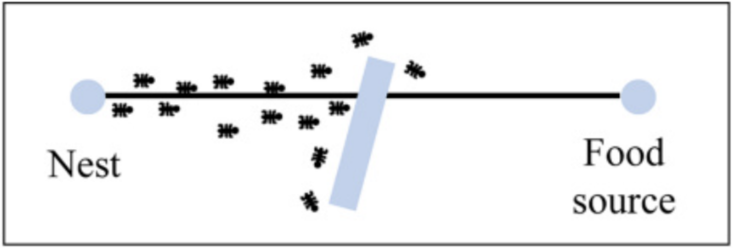

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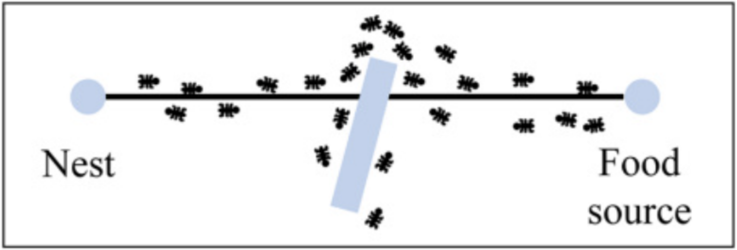

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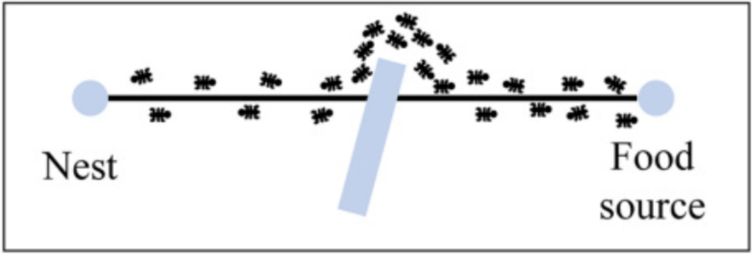

Ant Colony Optimization

Key Mechanisms

- The pheromone trails τ_{ij} guide the ants.
- Each ant constructs a complete solution by selecting components from the feasible set $\mathcal{N}_i$.
- Multiple ants explore different paths, and the algorithm identifies the path with the minimum cost.

Carbon Model for Vehicles

- **Transportation** accounts for 23% of the world's carbon emissions.

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- Emissions (in kgCO_2) depend on various factors:
  - Surface condition of the traveled route
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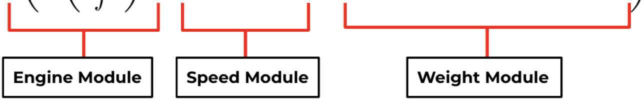
  - Weight load
  - Speed, etc.
- It's important to consider these factors in our model.

$$C(i,j) = \lambda u \left(y \left(\frac{d_{ij}}{f} \right) + \gamma^k \beta^k d_{ij} f^2 + \gamma^k s \left(\mu^k + F_{i,j,k,p,t} \right) d_{ij} \right)$$



Algorithm Overview: Stages of the *CAACS* Approach

- **Stage I: Finding a Path** — A valid GTSP solution is found.

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- **Stage I: Finding a Path** — A valid GTSP solution is found.
- **Stage II: Updating the Graph** — Adjusting the pheromone concentration based on the paths taken.
- **Representation of Nodes and Clusters**
  - Nodes in the GTSP graph are represented as 2×2 diamonds on a grid.
  - Each color represents a different cluster.

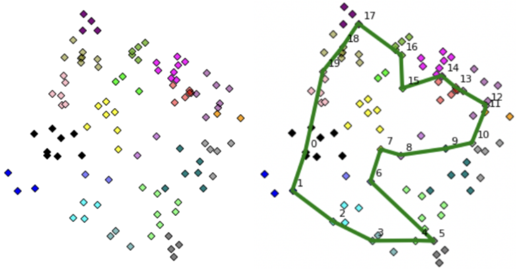

Figure 1: Example graph with a valid GTSP solution.

A Carbon Aware Ant Colony System

To simultaneously minimize cost and emissions, we introduce a novel emission hyperparameter E_{ij}.

$$E_{ij} = A^{1-\frac{c(i,j)}{c_{\max}}}$$

where c represents the carbon matrix, and

- $c(i,j)$: Carbon emission associated with path (i,j).
- $c_{\max}$: Maximum carbon emission among all possible paths.
- A: Scaling factor that adjusts the influence of emissions on path selection.

Stage I: Finding a Path

Exploitation: An ant moving from node i to node j follows:

$$j = \begin{cases} \arg\max_{u \in \mathcal{N}_i(t)} \{\tau_{iu}(t)\eta_{iu}(t)E_{iu}(t)\} & \text{if } r \leq r_0 \\ J & \text{otherwise.} \end{cases}$$

Weighted Evaluation of Edge

Cost Heuristic

CO_2 Heuristic

Risk Parameter

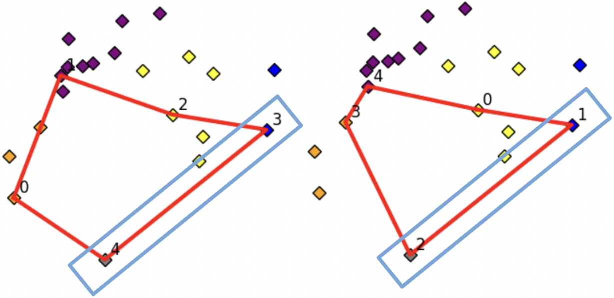

Figure 2: Example of Exploitation.

Stage I: Finding a Path

Exploration: If $r > r_0$, the node $J \in \mathcal{N}_i(t)$, is selected via the probability:

$$p_{iJ}(t) = \begin{cases} \dfrac{\tau_{iJ}(t)\eta_{iJ}(t)E_{iJ}(t)}{\sum_{u \in \mathcal{N}_i(t)} \tau_{iu}(t)\eta_{iu}(t)E_{iu}(t)} & \text{if } J \in \mathcal{N}_i(t) \\ 0 & \text{otherwise.} \end{cases}$$

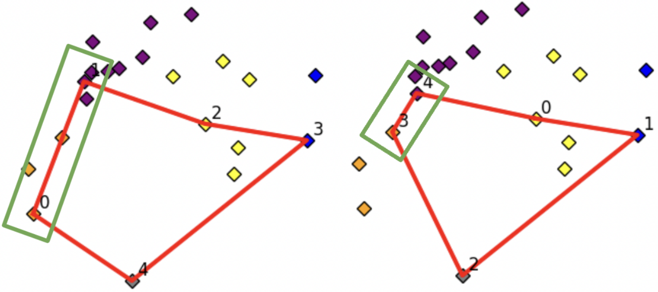

Figure 3: Example of Exploration.

Stage II: Updating the Graph

Carbon Aware Local Update Rule: The local update rule is applied immediately after an ant traverses an edge and is given by:

$$\tau_{ij}(t) \leftarrow \underbrace{(1 - \rho_L)\tau_{ij}(t)}_{\text{Reduction}} + \underbrace{\rho_L \tau_0 E_{ij}}_{\text{Reinforcement}}$$

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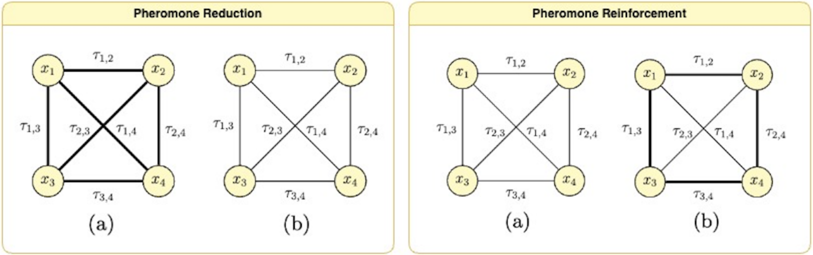
- Encourages exploration by reducing the pheromone level on the recently used edge and adds carbon aware reinforcement.



Stage II: Updating the Graph

Carbon Aware Global Update Rule: The global update rule is applied after all ants have completed their tours and is given by:

$$\tau_{ij}(t+1) = \underbrace{(1-\rho_G)\tau_{ij}(t)}_{\text{Reduction}} + \underbrace{\rho_G \overbrace{\Delta\tau_{ij}(t)}^{\text{Shortest Cost Heuristic}} E_{ij}}_{\text{Reinforcement}}$$

Stage II: Updating the Graph

Carbon Aware Global Update Rule: The global update rule is applied after all ants have completed their tours and is given by:

Shortest Cost Heuristic

$$\tau_{ij}(t+1) = (1-\rho_G)\tau_{ij}(t) + \rho_G \Delta\tau_{ij}(t) E_{ij}$$

Reduction

Reinforcement

- Reinforces the best solutions found so far for future iterations.

Evolution of Path Discovery

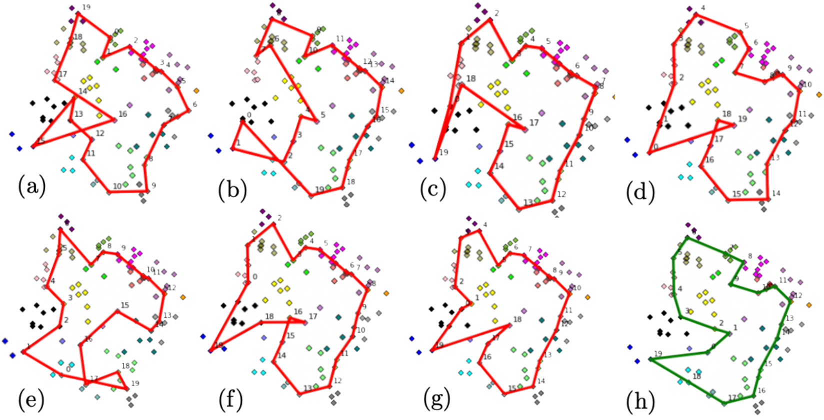


Figure 4: Illustration of Path Discovery.

Time Complexity

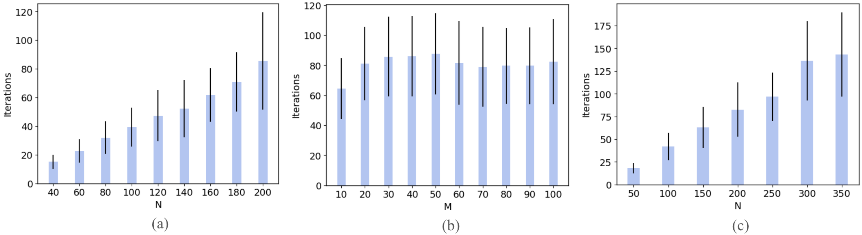


Figure 5: (a) Number of Nodes. (b) Number of Clusters. (c) Normalization.

Results: Empirically showed *linear time complexity*.

Number of Ants

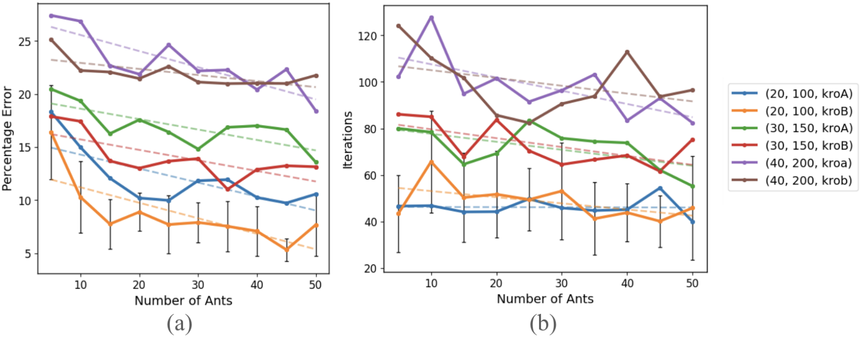


Figure 6: (a) **Quality of Solution:** Percentage error in cost compared to the optimal solution as the number of ants increases. (b) **Runtime:** Number of Iterations as the number of ants increases.

Application: Sustainable Delivery

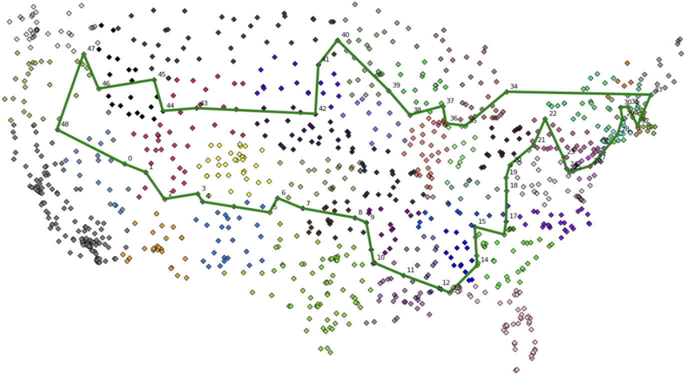


Figure 7: The final path generated by the algorithm on a grid with 1097 cities in 48 continental U.S. states and the District of Colombia (DC).

Acknowledgments

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- Dr. Etingof, Dr. Gerovitch, Dr. Khovanova, and the MIT PRIMES-USA organizers for making this amazing math research opportunity possible!

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